Module 1 Exercise

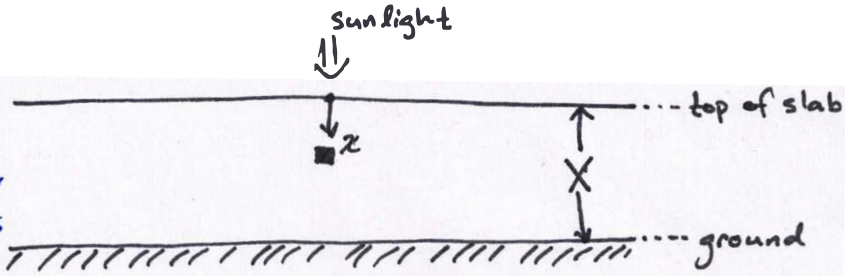
Corrections

ChE 330

Exercise 1.1: Light absorption

Step 1: define geometry & coordinate system

we need to choose this location for the x origin and not the ground in order to avoid transforming variables in expression $q = I_0 \varsigma e^{-\varsigma x}$

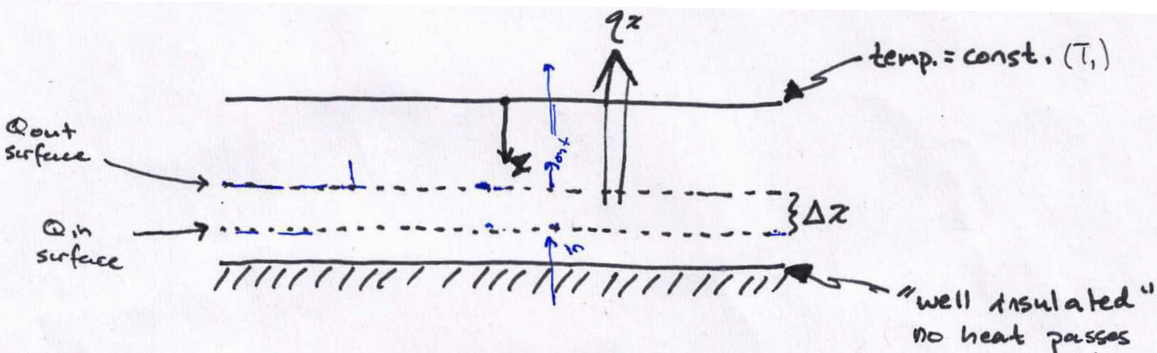


Note: we choose origin to be @ top of slab because our generation term is defined as a $f(x)$.

Step 2: we want to know $T(x)$

we assume $T \neq T(y, z, t)$ (steady state)

Step 3:



Heat balance eq.

$$Q_{in} - Q_{out} + Q_{prod} = 0$$

Note: Since bottom of slab is "well insulated" heat can only escape system at the top ($x=0$) thus we expect overall heat flow in the negative x direction. i.e. we expect q_x to be negative!

$$+Q_{in} \Rightarrow -Aq|_{x+\Delta x}$$

$$-Q_{out} \Rightarrow Aq|_x$$

then our heat balance becomes:

$$-\underbrace{Aq|_{x+\Delta x}}_{q_{in}} + \underbrace{Aq|_x}_{q_{out}} + \underbrace{A\Delta x}_{volume} I_0 \varsigma e^{-\varsigma x} = 0$$

Rearrange:

$$\frac{q|_{x+\Delta x} - q|_x}{\Delta x} = I_0 \varsigma e^{-\varsigma x} \xrightarrow[\Delta x \to 0]{\text{take limit}} \frac{dq}{dx} = I_0 \varsigma e^{-\varsigma x}$$

Step 4: boundary conditions

$$B.C.\,1 \Rightarrow T(x=0)=T_1 \qquad B.C.\,2 \Rightarrow q(x=\dot{X})=0$$

Step 5: solve

separate & integrate

$$\int dq = \int I_0 S e^{-Sx}\,dx \xrightarrow{\text{u substitution}} q = -I_0 e^{-Sx} + C_1$$

apply B.C. 2

$$0 = -I_0 e^{-Sx} + C_1 \rightarrow C_1 = +I_0 e^{-S\dot{X}}$$

so

$$q = -I_0\left[e^{-Sx} - e^{-S\dot{X}}\right]$$

or

$$q = I_0\left[e^{-S\dot{X}} - e^{-Sx}\right]$$

apply Fourrier's 1st law $\left(-k\frac{dT}{dx}=q\right)$

$$\frac{dT}{dx} = \frac{I_0}{k}\left[e^{-Sx} - e^{-S\dot{X}}\right]$$

separate + integrate

$$\int dT = \int \frac{I_0}{k}\left[e^{-Sx} - e^{-S\dot{X}}\right]dx \rightarrow T = \frac{I_0}{k}\left[-\frac{1}{S}e^{-Sx} - xe^{-S\dot{X}}\right] + C_2$$

apply B.C. 1 $T|_{x=0} = T_1$

$$T_1 = \frac{I_0}{k}\left[-\frac{1}{S} - 0\right] + C_2 \rightarrow C_2 = T_1 + \frac{I_0}{kS}$$

plussing in and simplifying

$$\boxed{T(x) = T_1 - \frac{I_0}{k}\left[\frac{1}{S}e^{-Sx} + xe^{-S\dot{X}}\right] + \frac{I_0}{kS}}$$

b) T_{max} should occur when $x = \dot{X}$, so

$$T_{max} = T_1 - \frac{I_0}{k}\left[\frac{1}{S}e^{-S\dot{X}} + \dot{X}e^{-S\dot{X}}\right] + \frac{I_0}{kS} \rightarrow T_{max} = T_1 - \frac{I_0}{k}e^{-S\dot{X}}\left[\frac{1}{S} + \dot{X}\right] + \frac{I_0}{kS}$$

Numerical evaluation

$$T_{max} = 20°C - \underbrace{\frac{1000}{0.53}\left[\exp(-124\cdot 0.1)\right]\left[\frac{1}{124} + 0.1\right]}_{\text{very small}} + \frac{1000}{0.53\cdot 124}$$

$$\boxed{T_{max} = 35°C}$$